UNIT-02

14/07/2025 **POISSON DISTRIBUTION** Monday

*Poisson Distribution:- Poisson Distribution was discovered by French Mathematician Simeon Denis poisson.

Definition: A discrete random variable X is said to follow poisson distribution with the parameter "λ" which assumes non-negative values and its probability mass function is given by.

$$P(X=x)=\begin{cases} \dfrac{e^{-\lambda}\lambda^{x}}{x!} & x=0,1,2,\dots \quad \lambda>0 \\ 0 & ;\ \text{otherwise} \end{cases}$$

Symbiocally it is denoted by $\boxed{X \text{ follows} \sim P(\lambda)}$

Examples: 1. No. of mistakes in a page of a book
2. No. of accidents in a month

*Moments of poisson distribution: we know that the r^{th} moment about origin is denoted by μ_r' is defined as $\mu_r' = E(x^r)$

$$\mu_r' = \Sigma x^r p(x)$$

If $r=1$

$$\mu_1' = \sum_{x=0}^{\infty} x\cdot p(x)$$

$$= \sum_{x=0}^{\infty} x \left(\frac{e^{-\lambda} \lambda^x}{x!} \right)$$

$$= e^{-\lambda} \sum_{x=0}^{\infty} x \frac{\lambda^x}{x!}$$

$$= e^{-\lambda} \sum_{x=0}^{\infty} \frac{\cancel{x} \lambda^{x-1} \lambda}{\cancel{x}(x-1)!}$$

$$= e^{-\lambda} \lambda \sum_{x=0}^{\infty} \frac{\lambda^{x-1}}{(x-1)!}$$

$$= e^{-\lambda} \lambda \left[\frac{\lambda^{1-1}}{(1-1)!} + \frac{\lambda^{2-1}}{(2-1)!} + \frac{\lambda^{3-1}}{(3-1)!} + \cdots \right]$$

$$= e^{-\lambda} \lambda \left[1 + \frac{\lambda}{1} + \frac{\lambda^2}{2!} + \cdots \right]$$

$$= \cancel{e^{-\lambda}} \lambda \left[\cancel{e^{\lambda}} \right] \Rightarrow \lambda$$

$$\boxed{\mu_1' = \lambda}$$ //

If $r = 2$

$$\mu_2' = \sum_{x=0}^{\infty} x^2 p(x)$$

$$= \sum_{x=0}^{\infty} (x(x-1) + x) \frac{e^{-\lambda} \lambda^x}{x!}$$

$$= \sum_{x=0}^{\infty} x(x-1) \frac{e^{-\lambda} \lambda^x}{x!} + \sum_{x=0}^{\infty} x \frac{e^{-\lambda} \lambda^x}{x!}$$

$$= e^{-\lambda} \sum_{x=2}^{\infty} \frac{\cancel{x(x-1)} \lambda^{x-2} \lambda^2}{\cancel{x(x-1)}(x-2)!} + \lambda$$

$$= e^{-\lambda} \lambda^2 \sum_{x=2}^{\infty} \frac{\lambda^{x-2}}{(x-2)!} + \lambda$$

$$= e^{-\lambda} \lambda^2 \left[\frac{\lambda^{2-2}}{(2-2)!} + \frac{\lambda^{3-2}}{(3-2)!} + \frac{\lambda^{4-2}}{(4-2)!} + \cdots \right] + \lambda$$

$$= e^{-\lambda}\lambda^2\left[1+\lambda+\frac{\lambda^2}{2!}+\cdots\right]+\lambda$$

$$= \cancel{e^{-\lambda}}\lambda^2(\cancel{e^{\lambda}})+\lambda$$

$$\boxed{\mu_2' = \lambda^2+\lambda}$$

* Central moments:- (i) $\boxed{\mu_1 = 0}$

(ii) $\mu_2 = \mu_2' - (\mu_1')^2$

$$= \lambda^2+\lambda-\lambda^2$$

$$\boxed{\mu_2 = \lambda}$$

(iii) $\mu_3 = \mu_3' - 3\mu_2\mu_1 + 2\mu_1^3$

$$= \lambda^3+3\lambda^2+\lambda-3(\lambda^2+\lambda)(\lambda)+2(\lambda^3)$$

$$= \cancel{\lambda^3}+\cancel{3\lambda^2}+\lambda-\cancel{3\lambda^3}-\cancel{3\lambda^2}+\cancel{2\lambda^3} = \lambda$$

$$\boxed{\mu_3 = \lambda}$$

(iv) $\mu_4 = \mu_4' - 4\mu_3'\mu_1' + 6\mu_2'(\mu_1')^2 - 3(\mu_1')^4$

$$= \lambda^4+6\lambda^3+7\lambda^2+\lambda-4(\lambda^3+3\lambda^2+\lambda)(\lambda)+6(\lambda^2+\lambda)\lambda^2-3\lambda^4$$

$$= \cancel{\lambda^4}+\cancel{6\lambda^3}+7\lambda^2+\lambda-\cancel{4\lambda^4}-\cancel{12\lambda^3}-4\lambda^2+\cancel{6\lambda^4}+\cancel{6\lambda^3}-\cancel{3\lambda^4}$$

$$\boxed{\mu_4 = 3\lambda^2+\lambda}$$

$$\beta_1 = \frac{\mu_3^2}{\mu_2^3} = \frac{\lambda^2}{\lambda^3} = \frac{1}{\lambda} \Rightarrow \boxed{\beta_1 = \frac{1}{\lambda}}$$

$$\beta_2 = \frac{\mu_4}{\mu_2^2} = \frac{3\lambda^2+\lambda}{\lambda^2} = \frac{3\lambda^2}{\lambda^2}+\frac{\lambda}{\lambda^2} = 3+\frac{1}{\lambda} \Rightarrow \boxed{\beta_2 = 3+\frac{1}{\lambda}}$$

Skewness:-

$$\gamma_1 = \sqrt{\beta_1} = \sqrt{\frac{1}{\lambda}} = \frac{1}{\sqrt{\lambda}} \Rightarrow \boxed{\gamma_1 = \frac{1}{\sqrt{\lambda}}}$$

$\gamma_2 = \beta_2 - 3 = 3 + \frac{1}{\lambda} - 3 = \frac{1}{\lambda} \Rightarrow \boxed{\gamma_2 = \frac{1}{\lambda}}$

Imp

* **M.G.F of poisson distribution:-**

we know that M.G.F is denoted by $M_X(t)$ and is defined as

$$M_X(t) = E[e^{tx}]$$

$$= \sum e^{tx} p(x)$$

$$= \sum_{x=0}^{\infty} e^{tx} \frac{e^{-\lambda} \lambda^x}{x!}$$

$$= e^{-\lambda} \sum_{x=0}^{\infty} \frac{(\lambda e^t)^x}{x!}$$

$$= e^{-\lambda} \left[\frac{(\lambda e^t)^0}{0!} + \frac{(\lambda e^t)^1}{1!} + \frac{(\lambda e^t)^2}{2!} + \cdots \right]$$

$$= e^{-\lambda} e^{\lambda e^t}$$

$$\boxed{M_X(t) = e^{\lambda(e^t - 1)}}$$

V. Imp

* **Moments through M.G.F:-** Mean and variance

we know that

$$\mu_r' = \left[\frac{d^r}{dt^r} M_X(t)\right]_{t=0}$$

if $r = 1$

$$\mu_1' = \left[\frac{d}{dt} M_X(t)\right]_{t=0}$$

$$= \left[\frac{d}{dt} e^{\lambda(e^t - 1)}\right]_{t=0}$$

$$\mu_1' = \left[e^{\lambda(e^t-1)} \lambda e^t\right]_{t=0}$$

$$= \left[e^{\lambda(e^0-1)} \lambda e^0\right] = \left[e^{\lambda(1-1)} \lambda\right] = [e^0 \lambda] = \lambda$$

$$\boxed{\mu_1' = \lambda}$$

if $r=2$

$$\mu_2' = \left[\frac{d}{dt} M_x'(t)\right]_{t=0}$$

$$= \left[\frac{d}{dt} e^{\lambda(e^t-1)} \lambda e^t\right]_{t=0}$$

$$= \lambda \left[\frac{d}{dt} e^{\lambda(e^t-1)} e^t\right]_{t=0}$$

$$= \lambda \left[e^{\lambda(e^t-1)} \lambda e^t \cdot e^t + e^{\lambda(e^t-1)} \cdot e^t\right]_{t=0}$$

$$= \lambda \left[e^{\lambda(e^t-1)} \lambda e^{2t} + e^{\lambda(e^t-1)} \cdot e^t\right]_{t=0}$$

$$= \lambda \left[e^{\lambda(e^0-1)} \lambda e^{0t} + e^{\lambda(e^0-1)} e^0\right]$$

$$= \lambda[\lambda+1]$$

$$\boxed{\mu_2' = \lambda^2 + \lambda}$$

Imp

* characteristics function of poisson distribution:

we know that CF is denoted $\phi_x(t)$ & is defined as $\phi_x(t) = E(e^{itx})$

$$= \Sigma e^{itx} p(x)$$

$$= \sum_{x=0}^{\infty} e^{itx} p(x)$$

$$= \sum_{x=0}^{\infty} e^{itx} \frac{e^{-\lambda}\lambda^x}{\lambda!}$$

$$\phi_x(t) = e^{-\lambda} \sum_{x=0}^{\infty} \frac{(\lambda e^{it})^x}{x!}$$

$$= e^{-\lambda}\left[\frac{(\lambda e^{it})^0}{0!} + \frac{(\lambda e^{it})^1}{1!} + \frac{(\lambda e^{it})^2}{2!} + \cdots\right]$$

$$= e^{-\lambda} e^{\lambda e^{it}}$$

$$= e^{\lambda (e^{it}-1)}$$

$$\boxed{\phi_X(t) = e^{\lambda (e^{it}-1)}}$$

22/07/2025 Tuesday

* C.G.F of poisson distribution :-

we know that C.G.F is denoted by $K_X(t)$ and is defined as $\log M_X(t)$

$$K_X(t) = [\log (M_X(t))]$$

$$= \log (e^{\lambda (e^t-1)})$$

$$= \lambda (e^t-1)$$

$$= \lambda \left(1 + t + \frac{t^2}{2!} + \frac{t^3}{3!} + \frac{t^4}{4!} + \cdots - 1\right)$$

$$K_X(t) = \cancel{\lambda} + \lambda \left[t + \frac{t^2}{2!} + \frac{t^3}{3!} + \frac{t^4}{4!} + \cdots\right) - \cancel{\lambda}$$

$$= \lambda \left(t + \frac{t^2}{2!} + \frac{t^3}{3!} + \frac{t^4}{4!} \cdots\right)$$

$$K_1 t + K_2 \frac{t^2}{2!} + K_3 \frac{t^3}{3!} + K_4 \frac{t^4}{4!} + \cdots$$

$$= \lambda \left(t + \frac{t^2}{2!} + \frac{t^3}{3!} + \frac{t^4}{4!} + \cdots\right)$$

Comparing coefficients of t, t^2, t^3, t^4 on both side

$$K_1 = \lambda(1) = \lambda = \mu_1'$$

$$\frac{K_2}{2!} = \frac{\lambda}{2!} \Rightarrow K_2 = \lambda = \text{Variance}$$

$$\frac{K_3}{3!} = \frac{\lambda}{3!} \Rightarrow K_3 = \lambda$$

$$\frac{K_4}{4!} = \frac{\lambda}{4!} \Rightarrow K_4 = \lambda$$

$\mu_4 = K_4 + 3K_2^2$

$= \lambda + 3\lambda^2$

$= 3\lambda^2 + \lambda$

Imp

* P.G.F of poisson Distribution:- we know that p.G.F is denoted by $P_X(s)$ and it is defined as $P_X(s) = E(s^x)$

$$= \sum_{x=0}^{\infty} s^x . p(x)$$

$$= \sum_{x=0}^{\infty} s^x \frac{e^{-\lambda}\lambda^x}{x!}$$

$$= \sum_{x=0}^{\infty} \frac{e^{-\lambda}(\lambda s)^x}{x!}$$

$$= e^{-\lambda} \sum_{x=0}^{\infty} \frac{(\lambda s)^x}{x!}$$

$$= e^{-\lambda}\left[\frac{(\lambda s)^0}{0!} + \frac{\lambda s^1}{1!} + \frac{\lambda s^2}{2!} \dots\right]$$

$$= e^{-\lambda}\left[1 + \lambda s + \frac{(\lambda s)^2}{2!} + \dots\right]$$

$P_X(s) = e^{-\lambda} e^{\lambda s} = e^{-\lambda + \lambda s} = e^{\lambda(s-1)}$

$$\boxed{P_X(s) = e^{\lambda(s-1)}}$$

* Recurrence relation for the moments of poisson distribution:- we know that r^{th} moment about mean is denoted by μ_r

and it is defined as

$$\mu_r = E[X - P(X)]^r$$

$$= E[X - \text{Mean}]^r$$

$$= E[X - \lambda]^r$$

$$= \sum_{x=0}^{\infty} (x-\lambda)^r p(x)$$

$$= \sum_{x=0}^{\infty} (x-\lambda)^r \frac{e^{-\lambda}\lambda^x}{x!}$$

differentiate with respect to λ

$$\frac{d}{d\lambda} \mu_r = \frac{d}{d\lambda}\left[\sum_{x=0}^{\infty} (x-\lambda)^r \cdot \frac{e^{-\lambda}\lambda^x}{x!}\right]$$

$$= \sum_{x=0}^{\infty}\left[\frac{d}{d\lambda}(x-\lambda)^r \frac{e^{-\lambda}\lambda^x}{x!}\right]$$

$$= \sum_{x=0}^{\infty}\left[r(x-\lambda)^{r-1}(-1)\frac{e^{-\lambda}\lambda^x}{x!} + (x-\lambda)^r\left[\frac{-e^{-\lambda}\lambda^x + e^{-\lambda}x\lambda^{x-1}}{x!}\right]\right]$$

$$= \sum_{x=0}^{\infty}\left[-r(x-\lambda)^{(r-1)}\frac{e^{-\lambda}\lambda^x}{x!} + (x-\lambda)^r \frac{e^{-\lambda}\lambda^x}{x!}\left(-1+\frac{x}{\lambda}\right)\right]$$

$$= \sum_{x=0}^{\infty} -r(x-\lambda)^{r-1}\frac{e^{-\lambda}\lambda^x}{x!} + \sum_{x=0}^{\infty}(x-\lambda)^r \frac{e^{-\lambda}\lambda^x}{x!}\left(\frac{x-\lambda}{\lambda}\right)$$

$$= \sum_{x=0}^{\infty} -r(x-\lambda)^{r-1}\frac{e^{-\lambda}\lambda^x}{x!} + \sum_{x=0}^{\infty}(x-\lambda)^{r+1}\frac{e^{-\lambda}\lambda^x}{x!}\left(\frac{1}{\lambda}\right)$$

$$= -r\sum_{0}^{\infty}(x-\lambda)^{r-1}\frac{e^{-\lambda}\lambda^x}{x!} + \frac{1}{\lambda}\sum_{x=0}^{\infty}(x-\lambda)^{r+1}\frac{e^{-\lambda}\lambda^x}{x!}$$

$$\frac{d\mu_r}{d\lambda} = -r\mu_{r-1} + \frac{1}{\lambda}\mu_{r+1}$$

$$\frac{1}{\lambda}\mu_{r+1} = \frac{d}{d\lambda}\mu_r + r\mu_{r-1}$$

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$$\boxed{\mu_{r+1} = \lambda\left[\frac{d}{d\lambda}\mu_r + r\mu_{r-1}\right]}$$ //

23/07/2025

wednesday

If $r=0$

$$\mu_{0+1} = \lambda\left[\frac{d}{d\lambda}\mu_0 + 0\mu_{0-1}\right]$$

$$= \lambda\left[\frac{d}{d\lambda}(1) + 0\right] \Rightarrow \boxed{\mu_1 = 0}$$ //

If $r=1$

$$\mu_2 = \lambda\left[\frac{d}{d\lambda}\mu_1 + (1)\mu_0\right]$$

$$= \lambda\left[\frac{d}{d\lambda}0 + 1\right]$$

$$= \lambda[0+1] = \lambda.$$

$$\boxed{\mu_2 = \lambda}$$ //

If $r=2$.

$$\mu_3 = \lambda\left[\frac{d}{d\lambda}\mu_2 + 2\mu_1\right] \Rightarrow \lambda\left[\frac{d}{d\lambda}(\lambda) + 0\right]$$

$$= \lambda[1+0]$$

$$\boxed{\mu_3 = \lambda}$$ //

If $r=3$

$$\mu_4 = \lambda\left[\frac{d}{d\lambda}\mu_3 + 3\mu_2\right]$$

$$= \lambda\left[\frac{d}{d\lambda}(\lambda) + 3\lambda\right]$$

$$= \lambda[1+3\lambda]$$

$$\boxed{\mu_4 = \lambda + 3\lambda^2}$$ //

Imp

Additive property of poisson distribution:-

Statement :- Sum of Independent poisson variates of a poisson variate.

proof :- let $x_1, x_2 \dots x_n$ be n independent poisson random variables with the parameters $\lambda_1, \lambda_2 \dots \lambda_n$ respectively.

we know that M.G.F of poisson distribution is

$$M_x(t) = e^{\lambda(e^t-1)}$$

Consider M.G.F of $x_1, x_2 \dots x_n$.

$$M_{x_1+x_2+\dots x_n}(t) = M_{x_1}(t) \cdot M_{x_2}(t) \cdots M_{x_n}(t)$$

$$= e^{\lambda_1(e^t-1)} \cdot e^{\lambda_2(e^t-1)} \cdots e^{\lambda_n(e^t-1)}$$

$$M_{\sum_{i=1}^{n} x}(t) = e^{\lambda_1(e^t-1) + \lambda_2(e^t-1) + \dots + \lambda_n(e^t-1)}$$

$$= e^{(e^t-1)[\lambda_1+\lambda_2+\dots+\lambda_n]}$$

$$M_{\sum_{i=1}^{n} x_i}(t) = e^{\sum_{i=1}^{n} \lambda_i(e^t-1)}$$

this is the M.G.F of poisson distribution.

Hence uniqueness ~~the~~ theorem of M.G.F

$\sum x_i$ follows poisson distribution with parameter $\sum \lambda_i$.

Imp

Recurrence relationship for the probability of poisson distribution :- we know that

$$P(X=x) = \frac{e^{-\lambda}\lambda^{x}}{x!} = p(x)$$

$$P(x+1) = \frac{e^{-\lambda}\lambda^{x+1}}{x+1!}$$

$$P(X=2) = \frac{e^{-\lambda}\lambda^{2}}{2!}$$

Consider $\frac{p(x+1)}{p(x)} = \frac{\frac{e^{-\lambda}\lambda^{x+1}}{x+1!}}{\frac{e^{-\lambda}\lambda^{x}}{x!}} = \frac{\cancel{e^{-\lambda}}\lambda^{x}\lambda}{x+1!} \times \frac{x!}{\cancel{e^{-\lambda}\lambda^{x}}}$

$$\frac{p(x+1)}{p(x)} = \frac{\lambda}{x+1}$$

$$\boxed{p(x+1) = p(x)\frac{\lambda}{x+1}}$$

V. Imp

*** Poisson distribution as a limiting case of Binomial distribution :-**

Binomial distribution tends to poisson distribution under following conditions.

1. No. of trails are indefinitely large, i.e. 'n' $\to \infty$
2. The probability of success p is small, i.e $p \to 0$.
3. $np = \lambda$ is finite, then $p = \frac{\lambda}{n}$

$q = 1-p \Rightarrow 1-\frac{\lambda}{n}$.

The P.M.F of Binomial distribution is

$$P(x) = \binom{n}{x} p^{x} q^{n-x}$$

$$= \frac{n!}{(n-x)!\, x!}\left(\frac{\lambda}{n}\right)^{x}\left(1-\frac{\lambda}{n}\right)^{n-x} \quad \left[\because {}^{n}C_{x} = \frac{n!}{(n-x)!\,x!}\right]$$

$$= \frac{n(n-1)(n-2)\cdots(n-(x-1))(n-x)!}{(n-x)!\,x!}\,\frac{\lambda^{x}}{n^{x}}\left(1-\frac{\lambda}{n}\right)^{n}\left(1-\frac{\lambda}{n}\right)^{-x}$$

$$= \frac{n(n-1)(n-2)\cdots(n-(x-1))}{x!}\,\frac{\lambda^{x}}{n^{x}}\left(1-\frac{\lambda}{n}\right)^{n}\frac{1}{\left(1-\frac{\lambda}{n}\right)^{x}}$$

$$= \frac{n^{x}\left[\left(1-\frac{0}{n}\right)\left(1-\frac{1}{n}\right)\left(1-\frac{2}{n}\right)\cdots\left(1-\left(\frac{x-1}{n}\right)\right)\right]}{x!}\;\frac{\lambda^{x}}{n^{x}}\left(1-\frac{\lambda}{n}\right)^{n}\frac{1}{\left(1-\frac{\lambda}{n}\right)^{x}}$$

As $n \to \infty$

$$= \frac{\lambda^{x}}{x!}\lim_{n\to\infty}\left(1-\frac{0}{n}\right)\left(1-\frac{1}{n}\right)\left(1-\frac{2}{n}\right)\cdots\left(1-\left(\frac{x-1}{n}\right)\right)\lim_{n\to\infty}\left(1-\frac{\lambda}{n}\right)^{n}\frac{1}{\lim_{n\to\infty}\left(1-\frac{\lambda}{n}\right)^{x}}$$

$$= \frac{\lambda^{x}}{x!}(1\cdot1\cdot1\cdots1)\,e^{-\lambda}\,\frac{1}{e^{0}}$$

$$\left\{\begin{array}{l}\lim_{n\to\infty}\left(1-\frac{\lambda}{n}\right)^{n} = e^{-\lambda}\\ \lim_{n\to\infty}\left(1-\frac{\lambda}{n}\right)^{x} = e^{0}\end{array}\right\}$$

$$\boxed{P(x) = \frac{\lambda^{x}}{x!}e^{-\lambda}}$$ //

*** MODE of P.D:-** Mode is the value of random variable for which the probability is maximum.

If $P(X=x)$ is maximum then $\left(P(x) = \frac{e^{-\lambda}\lambda^{x}}{x!}\right)$

$$P(X=x-1) \le P(x) \ge P(X=x+1)$$

$$\frac{e^{-\lambda}\lambda^{x-1}}{(x-1)!} \le \frac{e^{-\lambda}\lambda^{x}}{x!} \ge \frac{e^{-\lambda}\lambda^{x+1}}{(x+1)!}$$

$$\frac{e^{-\lambda}\lambda^{x}\lambda^{-1}}{(x-1)!} \leq \frac{e^{-\lambda}\lambda^{x}}{x(x-1)!} \geq \frac{e^{-\lambda}\lambda^{x}\lambda}{(x+1)(x(x-1)}$$

$$\frac{1}{\lambda} \leq \frac{1}{x} \geq \frac{1}{x(x+1)}$$

multiplying with x

$$\frac{x}{\lambda} \leq 1 \geq \frac{\lambda}{x+1}$$

Case-(i):- $\frac{x}{\lambda} \leq 1$

$x \leq \lambda$ — ①

Case-(ii):- $1 \geq \frac{\lambda}{x+1}$

$x+1 \geq \lambda$

$x \geq \lambda-1$ — ②

from ① and ②

$$\lambda-1 \leq x \leq \lambda$$

Case-(i):- If λ is a positive integer and since the integer x lies between two consecutive integers $\lambda-1$ and λ and this is possible only iff $x = \lambda-1$ and $x=\lambda$.

In this case the distribution is biomodel

Case (ii):- If λ is not an integer, then the integer x lies between two fractions differs from ①,② ~~to~~ one ~~to~~ another. Hence x, the mode must be the integral part of λ.

problem :-

1. In a poisson distribution, if $p(X=0) = 2p(X=1)$ then find $p(X=2)$

Sol $p(x) = \frac{e^{-\lambda}\lambda^{x}}{x!}$

$p(X=0) = 2p(X=1)$

$\frac{e^{-\lambda}\lambda^{0}}{0!} = 2\frac{e^{-\lambda}\lambda^{1}}{1!}$

$\Rightarrow 1 = 2\lambda$

$\Rightarrow \lambda = 1/2$

$\lambda = 0.5$

$p(X=2) = \frac{e^{-\lambda}\lambda^{2}}{2!}$

$= \frac{e^{-0.5}(0.5)^2}{2}$

$= \frac{(0.6065)(0.25)}{2}$

$p(X=2) = 0.075$

2. In a poisson distribution, if $\frac{3}{2}p(X=1) = p(X=3)$ then find (i) $p(X \geq 1)$

(ii) $p(2 \leq x \leq 5)$

Sol $p(x) = \frac{e^{-\lambda}\lambda^{x}}{x!}$

$\frac{3}{2}p(X=1) = p(X=3)$

$\frac{3}{2}\frac{e^{-\lambda}\lambda^{1}}{1!} = \frac{e^{-\lambda}\lambda^{3}}{3!}$

$\frac{3}{2}\lambda = \frac{\lambda^3}{6}$

$\frac{18}{2} = \lambda^2$

$\lambda^2 = 9$

$\boxed{\lambda = 3}$

(i) $p(X \geq 1) = 1 - p(X < 1)$

$= 1 - p(X=0)$

$= 1 - \frac{e^{-\lambda}\lambda^{0}}{0!}$

$= 1 - \frac{e^{-3}3^{0}}{0!}$

$= 1 - 0.0497$

$= 0.9502$

(ii) $p(2 \le x \le 5) = p(X=2) + p(X=3) + p(X=4) + p(X=5)$

$$= \frac{e^{-\lambda}\lambda^2}{2!} + \frac{e^{-\lambda}\lambda^3}{3!} + \frac{e^{-\lambda}\lambda^4}{4!} + \frac{e^{-\lambda}\lambda^5}{5!}$$

$$= \frac{e^{-3}3^2}{2!} + \frac{e^{-3}3^3}{3!} + \frac{e^{-4}4^4}{4!} + \frac{e^{-3}3^5}{5!}$$

$$= e^{-3}\left[\frac{9}{2} + \frac{27}{6} + \frac{81}{24} + \frac{243}{120}\right]$$

$$= e^{-3}[14.4]$$

$$= 0.7169$$

5M Imp

3. A book contains 43 mistakes out of 585 pages. Find the probability that there will be no mistake when 10 pages are selected randomly.

Sol: Given that $n = 10$

$$P = \frac{43}{585}$$

$$np = \lambda$$

$$\lambda = 10\left(\frac{43}{585}\right)$$

$$\lambda = \frac{430}{585}$$

$$\boxed{\lambda = 0.735}$$

$$P(X=0) = \frac{e^{-\lambda}\lambda^0}{0!}$$

$$= \frac{e^{-0.735}(0.735)^0}{0!} = 0.4795 = 0.481$$

$$= 0.481$$

Sraddha 25/07

* limiting case of poisson distribution to normal distribution:

"poisson distribution tends to normal distribution for large values of λ"?

m.g.f of standard poisson variates $\frac{X-\lambda}{\sqrt{\lambda}}$ approaches to $e^{t^2/2}$ as $\lambda \to \infty$

proof: let $X \sim P(\lambda)$, then consider standard poisson variate $Z = \frac{X-\lambda}{\sqrt{\lambda}}$. MGF is $e^{\lambda(e^t-1)}$

the m.g.f of Z is

$$M_Z(t) = E(e^{tZ}) = E\left(e^{t\left(\frac{X-\lambda}{\sqrt{\lambda}}\right)}\right)$$

$$= e^{-\frac{t\lambda}{\sqrt{\lambda}}} \cdot E\left(e^{\frac{tX}{\sqrt{\lambda}}}\right)$$

$$= e^{-t\sqrt{\lambda}} \cdot M_X(t/\sqrt{\lambda})$$

$$= e^{-t\sqrt{\lambda}} \cdot e^{\lambda(e^{t/\sqrt{\lambda}}-1)}$$

$$= e^{-t\sqrt{\lambda}+\lambda\left(e^{\frac{t}{\sqrt{\lambda}}}-1\right)}$$

consider logarithms

$$\log M_Z(t) = -t\sqrt{\lambda} + \lambda\left(e^{\frac{t}{\sqrt{\lambda}}}-1\right)$$

$$= -t\sqrt{\lambda} + \lambda\left[\left(1+\frac{t}{\sqrt{\lambda}}+\frac{t^2}{2!\lambda}+\frac{t^3}{3!\lambda^{3/2}}+\cdots\right)-1\right]$$

$$= -t\sqrt{\lambda} + \lambda\left[\frac{t}{\sqrt{\lambda}} + \frac{t^2}{2\lambda} + \frac{t^2}{3!\,\lambda^{3/2}} + \cdots\right]$$

$$= -t\sqrt{\lambda} + t\sqrt{\lambda} + \frac{t^2}{2} + O(\lambda^{-1/2})$$

$= \frac{t^2}{2} + O(\lambda^{-1/2})$

where $O(\lambda^{-1/2})$ is the terms containing the powers $\frac{1}{2}$ and more of λ in the denominator.

Now apply limits as $n \to \infty$

$$\lim_{n \to \infty} \log M_Z(t) = \frac{t^2}{2}$$

consider exponential on both sides

$$\lim_{n \to \infty} M_Z(t) = e^{t^2/2}$$

which is m.g.f of standard normal variate. Hence by uniqueness theorem of m.g.f standard poisson variate approaches standard normal distribution i.e. poisson distribution tends to normal distribution as $\lambda \to \infty$.//